

# Relating the generating functionals in field/antifield formulation through finite field dependent BRST transformation

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We study the field/antifield formulation of pure Yang Mills theory in the framework of finite field dependent BRST transformation. We show that the generating functionals corresponding to different solutions of quantum master equation are connected through the finite field dependent BRST transformations. We establish this result with the help of several explicit examples.

## I. INTRODUCTION

BRST symmetries are extremely useful in quantum field theory and play important role in the discussion of quantization, renormalization, unitarity and other aspects of gauge theories [1, 2, 3]. The nilpotent BRST transformation is characterized by an infinitesimal, anticommuting and space time independent parameter and leaves the effective action invariant. Joglekar and Mandal [4] generalized the BRST transformation where the parameter involved is finite and field dependent but space-time independent. Such finite field dependent BRST (FFBRST) transformations are also nilpotent and leave the Faddeev-Popov (FP) effective action invariant. However, the path integral measure changes in a non-trivial way due to the finite field dependent parameter in such transformations. It has been shown [4] that the non-trivial Jacobian of the path integral measure can always be expressed as  $e^{iS_1}$ , where  $S_1$  is some local function of field variables and can be a part

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of the effective action. Thus, FFBRST can connect the generating functionals of two different effective field theories with suitable choice of finite field dependent parameter [4]. For example, these can be used to connect FP effective action in Lorentz gauge with a gauge parameter  $\lambda$  to (i) the most general BRST/anti-BRST symmetric action in Lorentz gauges [4], (ii) the FP effective action in axial gauge [5], (iii) the FP effective action in Coulomb gauge [6], (iv) FP effective action with another distinct gauge parameter  $\lambda'$  [4] and (v) the FP effective action in quadratic gauge [4]. The choice of the parameter is crucial in connecting different effective gauge theories by means of the FFBRST.

FFBRST transformations have found many applications [5, 6, 7, 8, 9, 10] in the study of gauge theories. Correct prescription for the poles in the gauge field propagators in non-covariant gauges have been derived by connecting effective theories in covariant gauges to the theories in non-covariant gauges by using FFBRST [6, 9]. The divergent energy integrals in Coulomb gauge are regularized by modifying time like propagator using FFBRST [6].

In this present work, we have extended the FFBRST formulation for the case of field/antifield formulation of pure Yang Mills (YM) theory. We have shown, the generating functionals in field/antifield formulation, corresponding to different solutions of quantum master equation [2, 3, 11] are related through FFBRST transformations. The choice of finite field dependent BRST parameter plays crucial role in relating the generating functionals. A particular choice of FFBRST parameter connects a pair of generating functionals. We consider several choice of FFBRST parameter to show the connection explicitly.

This paper is organised as follows. We provide brief introduction to field/antifield formulation in Sec. 2.1 and a brief review of the FFBRST formulation in Sec. 2.2. In Sec. 3, we develop FFBRST transformation in the auxiliary field formulation which will be required in the later sections. We consider several choice of FFBRST parameter to establish the connection between the different generating functionals in field/antifield formulation in Sec. 4. In Sec. 4.1, we consider the connection of generating functionals for different effective theories in different gauges. Connection with most general BRST/anti-

BRST invariant theory is established in Sec. 4.2. Sec. 5 is devoted for summary and discussion.

## II. PRELIMINARY REVIEW

### A. Field/Antifield formulation

The Lagrangian quantization of Batalin & Vilkovisky [11], also known as field/antifield formulation is considered to be one of the most powerful procedures of quantization of gauge theories involving BRST symmetry [2, 3, 12, 13, 14, 15]. The main idea is to construct an extended action  $W_\Psi(\phi, \phi^*)$  by introducing antifields  $\phi^*$  corresponding to each field  $\phi$  with opposite statistic. Generically  $\phi$  denotes all the fields involved in the theory. The sum of ghost number associated to a field and its antifield is equal to -1. The generating functional can be written as

$$Z = \int D\phi e^{iW_\Psi[\phi]}, \quad (2.1)$$

where

$$W_\Psi[\phi] = W \left[ \phi, \phi^* = \frac{\partial \Psi}{\partial \phi} \right]. \quad (2.2)$$

$\Psi$  is the gauge fixed fermion and has grassman parity 1 and ghost number -1. The generating functional  $Z$  does not depend on the choice of  $\Psi$ . This extended quantum action satisfies certain rich mathematical relation called quantum master equation [3] and is given by

$$\Delta e^{iW_\Psi[\phi, \phi^*]} = 0 \quad \text{with} \quad \Delta \equiv \frac{\partial_r}{\partial \phi} \frac{\partial_r}{\partial \phi^*} (-1)^{\epsilon+1}. \quad (2.3)$$

Master equation reflects the gauge symmetry in the zeroth order of antifields and in the first order of antifields it reflects nilpotency of BRST transformation. This equation can also be written in terms of antibrackets as

$$(W_\Psi, W_\Psi) = 2i\Delta W_\Psi, \quad (2.4)$$

where the antibracket is defined as

$$(X, Y) \equiv \frac{\partial_r X}{\partial \phi} \frac{\partial_l Y}{\partial \phi^*} - \frac{\partial_r X}{\partial \phi^*} \frac{\partial_l Y}{\partial \phi}. \quad (2.5)$$

Different effective actions belonging to the same theory are solutions of master equations. The solutions of master equation are unique upto (anti-) canonical transformations which preserve the equation (2.4). For example, the solution of master equation corresponding to the extended action in axial gauge  $W_{\Psi}^A[\phi, \phi^*]$  is related to the solution of master equation corresponding to the extended action in Lorentz gauge  $W_{\Psi}^L[\phi, \phi^*]$  through (anti)-canonical transformation. In this paper, we show that the generating functionals corresponding to different solutions of master equation can be related through FFBRST transformation. In particular, we consider field/antifield formulation of pure YM theory to show the connection by using FFBRST transformation which will be reviewed briefly below.

## B. Finite field dependent BRST

Let us now briefly review the FFBRST approach [4, 5, 7, 9]. FFBRST transformations are obtained by an integration of infinitesimal (field dependent) BRST transformations [4]. In this method all the fields are functions of some parameter,  $\kappa : 0 \leq \kappa \leq 1$ . For a generic field  $\phi(x, \kappa)$ ,  $\phi(x, \kappa = 0) = \phi(x)$ , is the initial field and  $\phi(x, \kappa = 1) = \phi'(x)$  is the transformed field. Then the infinitesimal field dependent BRST transformations are defined as

$$\frac{d}{d\kappa} \phi(x, \kappa) = \delta_{BRST} \phi(x, \kappa) \Theta'[\phi(x, \kappa)], \quad (2.6)$$

where  $\Theta' d\kappa$  is an infinitesimal field dependent parameter. It has been shown [4] by integrating these equations from  $\kappa = 0$  to  $\kappa = 1$  that  $\phi'(x)$  are related to  $\phi(x)$  by FFBRST transformations

$$\phi'(x) = \phi(x) + \delta_{BRST} \phi(x) \Theta[\phi(x)], \quad (2.7)$$

where  $\Theta[\phi(x)]$  is obtained from  $\Theta'[\phi(x)]$  through the relation

$$\Theta[\phi(x)] = \Theta'[\phi(x)] \frac{\exp f[\phi(x)] - 1}{f[\phi(x)]}, \quad (2.8)$$

and  $f$  is given by  $f = \sum_i \frac{\delta \Theta'(x)}{\delta \phi_i(x)} \delta_{BRST} \phi_i(x)$ . These transformations are nilpotent and symmetry of the FP effective action. However, Jacobian of the path integral measure

changes the generating functional corresponding to FP effective theory to the generating functional for a different effective theory.

The meaning of these field transformations is as follows. We consider the vacuum expectation value of a gauge invariant functional  $G[A]$  in some effective theory,

$$\langle\langle G[A] \rangle\rangle \equiv \int \mathcal{D}\phi \, G[A] \exp(iS_{eff}[\phi]), \quad (2.9)$$

where

$$S_{eff} = S_0 + S_{gf} + S_g. \quad (2.10)$$

Here,  $S_0$  is the pure YM action

$$S_0 = \int d^4x \left[ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha \right], \quad (2.11)$$

and the gauge fixing and ghost part of the effective action in Lorentz gauge are given as

$$\begin{aligned} S_{gf} &= -\frac{1}{2\lambda} \int d^4x (\partial \cdot A^\alpha)^2, \\ S_g &= -\int d^4x \left[ \bar{c}^\alpha \partial^\mu D_\mu^{\alpha\beta} c^\beta \right]. \end{aligned} \quad (2.12)$$

The covariant derivative is defined as  $D_\mu^{ab}[A] \equiv \delta^{ab} \partial_\mu + g f^{abc} A_\mu^c$ .

Now we perform the FFBRST transformation  $\phi \rightarrow \phi'$  given by Eq. (2.7). Then we have

$$\langle\langle G[A] \rangle\rangle = \langle\langle G[A'] \rangle\rangle = \int \mathcal{D}\phi' J[\phi'] G[A'] \exp(iS_{eff}^F[\phi']), \quad (2.13)$$

on account of BRST invariance of  $S_{eff}$  and the gauge invariance of  $G[A]$ . Here  $J[\phi']$  is the Jacobian associated with FFBRST transformation and is defined as

$$\mathcal{D}\phi = \mathcal{D}\phi' J[\phi']. \quad (2.14)$$

Note that unlike the usual infinitesimal BRST transformation the Jacobian for FFBRST is not unity. In fact, this non-trivial Jacobian is the source of the new results in this formulation. As shown in Ref. [4] for the special case  $G[A] = 1$ , the Jacobian  $J[\phi']$  can always be replaced by  $\exp(iS_1[\phi'])$ , where  $S_1(\phi')$  is some *local* functional of the fields and can be added to action,

$$S_{eff}[\phi'] + S_1[\phi'] = S'_{eff}[\phi']. \quad (2.15)$$

Thus the FFBRST in Eq. (2.7) takes the generating functional with effective action  $S_{eff}[\phi]$  to the generating functional corresponding to another effective action  $S'_{eff}[\phi]$ . The extra part,  $S_1(\phi)$  of the new effective action  $S'_{eff}[\phi]$  depends on the choice of the parameter in FFBRST transformation. Thus, by choosing the parameter in FFBRST transformation appropriately one can connect the generating functionals corresponding to any two effective gauge theories. In particular the FFBRST of Eq. (2.7) with  $\Theta'[\phi(x, \kappa)] = i \int d^4y \bar{c}^\alpha(y, \kappa) [F^\alpha[A(y, \kappa)] - F'^\alpha[A(y, \kappa)]]$  relates the YM theory with an arbitrary gauge fixing  $F[A]$  to the YM theory with another arbitrary gauge fixing  $F'[A]$  [7].

### III. THE FFBRST IN AUXILIARY FIELD FORMULATION

To study the role of FFBRST in the field/antifield formulation, it is convenient to use auxiliary field (B) formulation [16]. In this section, we intend to generalize the FFBRST formulation in B field formulation. We only mention the necessary modifications of FFBRST formulation in presence of auxiliary field [18]. For simplicity, we consider the case of pure YM theory described by the action

$$S_{eff} = \int d^4x \left[ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + \frac{\lambda}{2} (B^\alpha)^2 - B^\alpha \partial \cdot A^\alpha - \bar{c}^\alpha \partial^\mu D_\mu^{\alpha\beta} c^\beta \right]. \quad (3.1)$$

Using the method outlined in Sec. 2.2 it is straightforward to find FFBRST transformations under which the above  $S_{eff}$  remains invariant. These transformations are as follows

$$\begin{aligned} A_\mu^\alpha &\rightarrow A_\mu^\alpha + D_\mu^{\alpha\beta} c^\beta \Theta(A, c, \bar{c}, B), \\ c^\alpha &\rightarrow c^\alpha - \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma \Theta(A, c, \bar{c}, B), \\ \bar{c}^\alpha &\rightarrow \bar{c}^\alpha + B^\alpha \Theta(A, c, \bar{c}, B), \\ B^\alpha &\rightarrow B^\alpha. \end{aligned} \quad (3.2)$$

However, non-trivial modification arises in the calculation of Jacobian for this FFBRST in auxiliary field formulation. The Jacobian can be defined as

$$DA(x)Dc(x)D\bar{c}(x)DB(x) = J(x, k)DA(x, k)Dc(x, k)D\bar{c}(x, k)DB(x, k)$$

$$= J(k+dk)DA(k+dk)Dc(k+dk)D\bar{c}(k+dk)DB(k+dk). \quad (3.3)$$

The transformation from  $\phi(k)$  to  $\phi(k+dk)$  is an infinitesimal one and one has, for its Jacobian

$$\frac{J(k)}{J(k+dk)} = \sum_{\phi} \pm \frac{\delta\phi(x, k+dk)}{\delta\phi(x, k)}, \quad (3.4)$$

where  $\sum_{\phi}$ , sums over all the fields in the measure  $A_{\mu}^{\alpha}, c^{\alpha}, \bar{c}^{\alpha}, B^{\alpha}$  and the  $\pm$  sign refers to if  $\phi$  is a bosonic or a fermionic field. We evaluate the right hand side as

$$\int d^4x \sum_{\alpha} \left[ \sum_{\mu} \frac{\delta A_{\mu}^{\alpha}(x, k+dk)}{\delta A_{\mu}^{\alpha}(x, k)} - \frac{\delta c^{\alpha}(x, k+dk)}{\delta c^{\alpha}(x, k)} - \frac{\delta \bar{c}^{\alpha}(x, k+dk)}{\delta \bar{c}^{\alpha}(x, k)} + \frac{\delta B^{\alpha}(x, k+dk)}{\delta B^{\alpha}(x, k)} \right], \quad (3.5)$$

dropping those terms which do not contribute on account of the antisymmetry of structure constant. We calculate infinitesimal Jacobian change as mentioned in [4] to be

$$\frac{1}{J(k)} \frac{dJ(k)}{dk} = - \int d^4x \left[ (\delta A_{\mu}^{\alpha}) \frac{\delta \Theta'}{\delta A_{\mu}^{\alpha}} - (\delta c^{\alpha}) \frac{\delta \Theta'}{\delta c^{\alpha}} - \delta \bar{c}^{\alpha} \frac{\delta \Theta'}{\delta \bar{c}^{\alpha}} \right], \quad (3.6)$$

since,  $\delta B^{\alpha} = 0$ . Further, it can be shown that the Jacobian in Eq. (3.3) can be expressed as  $e^{iS_1[\phi]}$  by following the general procedure mentioned in Ref [4]. To illustrate the procedure, we consider a simple example of FFBRST in auxiliary field formulation. Let us take  $\Theta'[\phi(y, k)] = i\gamma \int d^4y \bar{c}^{\alpha}(y, k) B^{\alpha}(y, k)$  where  $\gamma$  is an arbitrary constant parameter and then using Eq. (3.6), we obtain

$$\frac{1}{J(k)} \frac{dJ(k)}{dk} = i\gamma \int d^4y [B^{\alpha}(y, k)]^2. \quad (3.7)$$

This Jacobian  $J(k)$  will be replaced by  $e^{iS_1(\phi)}$  iff [4]

$$\int D\phi(k) \exp[iS_1 + iS_{eff}] \left[ \frac{1}{J(k)} \frac{dJ}{dk} - i \frac{dS_1}{dk} \right] = 0. \quad (3.8)$$

We make an ansatz,  $S_1 = \xi_1(k) \int d^4x [B^{\alpha}(x, k)]^2$  where  $\xi_1(k)$  is some  $\kappa$ -dependent arbitrary parameter satisfying the initial condition  $\xi_1(k=0) = 0$ . Now the condition in Eq. (3.8) is satisfied only when

$$i(\gamma - \xi_1'(k)) = 0. \quad (3.9)$$

By solving Eq. (3.9), we get  $\xi_1 = \gamma k$  and the extra term in the net effective action is

$$S_1(k=1) = \gamma \int d^4x [B^\alpha(x)]^2. \quad (3.10)$$

The effective action in Eq. (3.1) is modified as

$$\begin{aligned} S_{eff} + S_1 &= \int d^4x \left[ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + \frac{\lambda}{2} (B^\alpha)^2 - B^\alpha \partial \cdot A^\alpha - \bar{c}^\alpha \partial^\mu D_\mu^{\alpha\beta} c^\beta + \gamma (B^\alpha)^2 \right] \\ &= \int d^4x \left[ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + \frac{\lambda'}{2} (B^\alpha)^2 - B^\alpha \partial \cdot A^\alpha - \bar{c}^\alpha \partial^\mu D_\mu^{\alpha\beta} c^\beta \right], \end{aligned} \quad (3.11)$$

where

$$\lambda' = \lambda + 2\gamma. \quad (3.12)$$

Thus, FFBRST with parameter  $\Theta'[\phi(y, k)] = i\gamma \int d^4y \bar{c}^\alpha(y, k) B^\alpha(y, k)$ , connects effective action with two different distinct gauge parameter  $\lambda$  and  $\lambda'$ .

We use this auxiliary field formulation of FFBRST directly in the following sections where we consider field/antifield formulation.

#### IV. FFBRST IN FIELD/ANTIFIELD FORMULATION

In this section, we use FFBRST transformation in field/antifield formulation to show that the different generating functionals corresponding to different solution of master equation are related through FFBRST. We consider few explicit examples to establish this result.

##### A. Connecting solutions of master equation in different gauges

###### Case I: Lorentz gauge to axial gauge

We start with the generating functional of YM theory in Lorentz gauge as

$$Z^L = \int D\phi \exp [i S_{eff}(A, c, \bar{c}, B)], \quad (4.1)$$



where  $S_{eff}(A, c, \bar{c}, B)$ , is given by Eq. (3.1). This generating functional can be expressed in field/antifield formulation as

$$Z^L = \int [dA dc d\bar{c} dB] \exp \left[ i \int d^4x \left\{ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + A^{\mu\alpha*} D_\mu^{\alpha\beta} c^\beta + c^{\alpha*} \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma + B^\alpha \bar{c}^{\alpha*} \right\} \right], \quad (4.2)$$

or, compactly

$$Z^L = \int D\phi \exp \left[ i S_0(\phi) + i \delta \Psi^L \frac{1}{\Lambda} \right] \equiv \int D\phi \exp \left[ i W_\Psi^L(\phi, \phi^*) \right],$$

where the gauge fixed fermion

$$\Psi^L = \int d^4x \bar{c}^\alpha \left[ \frac{\lambda}{2} B^\alpha - \partial \cdot A^\alpha \right]. \quad (4.3)$$

$\delta \Psi^L$  is the BRST variation of  $\Psi^L$  and  $\Lambda$  is the infinitesimal, anticommuting parameter of the usual BRST transformation. The antifields  $A_\mu^{\alpha*}, c^{\alpha*}, \bar{c}^{\alpha*}, B^{\alpha*}$  corresponding to the fields  $A_\mu^\alpha, c^\alpha, \bar{c}^\alpha, B^\alpha$  are obtainable from the gauge fixed fermion,  $\Psi^L$  as

$$\begin{aligned} A^{\mu\alpha*} &= \frac{\delta \Psi^L}{\delta A_\mu^\alpha} = \partial^\mu \bar{c}^\alpha, \\ \bar{c}^{\alpha*} &= \frac{\delta \Psi^L}{\delta \bar{c}^\alpha} = \left[ \frac{\lambda}{2} B^\alpha - \partial \cdot A^\alpha \right], \\ c^{\alpha*} &= \frac{\delta \Psi^L}{\delta c^\alpha} = 0, \\ B^{\alpha*} &= \frac{\delta \Psi^L}{\delta B^\alpha} = \frac{\lambda}{2} \bar{c}^\alpha. \end{aligned} \quad (4.4)$$

Now, we apply the FFBRST transformation given by Eq. (3.2), to the above  $Z^L$  where  $\Theta(A, c, \bar{c}, B)$  is chosen to have a particular form, obtainable from

$$\Theta'(A, c, \bar{c}, B) = i \int d^4y \bar{c}^\alpha [\gamma_1 \lambda B^\alpha + (\partial \cdot A^\alpha - \eta \cdot A^\alpha)], \quad (4.5)$$

using Eq.(8).  $\gamma_1$  is an arbitrary constant. We calculate the Jacobian by following the same procedure as discussed in section 3, which can be expressed as  $e^{iS_1(\phi)}$  where

$$S_1 = \int d^4x \left[ \gamma_1 \lambda (B^\alpha)^2 + B^\alpha (\partial \cdot A^\alpha - \eta \cdot A^\alpha) - \bar{c}^\alpha (\eta^\mu - \partial^\mu) D_\mu^{\alpha\beta} c^\beta \right]. \quad (4.6)$$

The transformed generating functional becomes

$$\begin{aligned} Z'^L &= \int D\phi \exp \left\{ i \left[ W_\Psi^L(\phi, \phi^*) + S_1 \right] \right\}, \\ &= \int D\phi \exp \left[ i W_\Psi^A(\phi, \phi^*) \right] \equiv Z^A. \end{aligned} \quad (4.7)$$

Under the FFBRST with parameter given in Eq. (4.5),  $Z^L$  transforms to  $Z^A$ , which can be written explicitly in field/antifield formulation as

$$Z^A = \int [dA d\bar{c} d\bar{c} dB] \exp \left[ i \int d^4x \left\{ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + \tilde{A}^{\mu\alpha*} D_\mu^{\alpha\beta} c^\beta + \tilde{c}^{\alpha*} \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma + B^\alpha \tilde{c}^{\alpha*} \right\} \right]. \quad (4.8)$$

In compact notation,

$$Z^A = \int D\phi \exp \left[ i S_0(\phi) + i \delta \Psi^A \frac{1}{\Lambda} \right], \quad (4.9)$$

where

$$\Psi^A = \int d^4x \bar{c}^\alpha \left[ \frac{\xi}{2} B^\alpha - \eta \cdot A^\alpha \right], \quad (4.10)$$

with

$$\begin{aligned} \tilde{A}^{\mu\alpha*} &= \frac{\delta \Psi^A}{\delta A_\mu^\alpha} = -\bar{c}^\alpha \eta^\mu, \\ \tilde{c}^{\alpha*} &= \frac{\delta \Psi^A}{\delta \bar{c}^\alpha} = \left[ \frac{\xi}{2} B^\alpha - \eta \cdot A^\alpha \right], \\ \tilde{c}^{\alpha*} &= \frac{\delta \Psi^A}{\delta c^\alpha} = 0, \\ \tilde{B}^{\alpha*} &= \frac{\delta \Psi^A}{\delta B^\alpha} = \frac{\xi}{2} \bar{c}^\alpha. \end{aligned} \quad (4.11)$$

$\xi = \lambda(1+2\gamma)$  is the gauge parameter in axial gauge. This  $Z^A$  is the generating functional of YM theory for axial gauge in field/antifield formulation. Thus, the FFBRST transformation given by Eq. (3.2) with the parameter given in Eq. (4.5) takes  $Z^L$  to  $Z^A$ . Both the extended actions in field/antifield formulation,  $W_\Psi^L(\phi, \phi^*)$  and  $W_\Psi^A(\phi, \tilde{\phi}^*)$  are the solutions of the master equation given in Eq. (2.4) and these are linked through FFBRST.

### Case II: Lorentz gauge to Coulomb gauge

Now, we consider another choice of FFBRST parameter  $\Theta(A, c, \bar{c}, B)$  corresponding to

$$\Theta'(A, c, \bar{c}, B) = i \int d^4y \bar{c}^\alpha [\gamma_1 \lambda B^\alpha + \partial_o A_0^\alpha], \quad (4.12)$$

and apply the FFBRST transformation to  $Z^L$ . We calculate the Jacobian for this trans-

formation which produces the additional term in the action

$$S_1 = \int d^4x \left[ \gamma_1 \lambda (B^\alpha)^2 + B^\alpha \partial_0 A_0^\alpha + \bar{c}^\alpha \partial^0 D_0^{\alpha\beta} c^\beta \right]. \quad (4.13)$$

This yields

$$Z'^L = \int D\phi \exp \left\{ i \left[ W_\Psi^L(\phi, \phi^*) + S_1 \right] \right\} \equiv Z^C, \quad (4.14)$$

where

$$Z^C = \int [dA d\bar{c} d\bar{c} dB] \exp \left[ i \int d^4x \left\{ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + \tilde{A}^{\mu\alpha*} D_\mu^{\alpha\beta} c^\beta + \tilde{c}^{\alpha*} \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma + B^\alpha \tilde{c}^{\alpha*} \right\} \right]. \quad (4.15)$$

Or, compactly

$$Z^C = \int D\phi \exp \left[ i S_0(\phi) + i \delta \Psi^C \frac{1}{\Lambda} \right] \equiv \int D\phi \exp \left[ i W_\Psi^C(\phi, \tilde{\phi}^*) \right], \quad (4.16)$$

where

$$\Psi^C = \int d^4x \bar{c}^\alpha \left[ \frac{\xi}{2} B^\alpha - \partial^i A_i^\alpha \right], \quad (4.17)$$

with

$$\begin{aligned} \tilde{A}^{i\alpha*} &= \frac{\delta \Psi^C}{\delta A_i^\alpha} = -\bar{c}^\alpha \partial^i, \quad \tilde{A}^{0\alpha*} = 0, \\ \tilde{\bar{c}}^{\alpha*} &= \frac{\delta \Psi^C}{\delta \bar{c}^\alpha} = \left[ \frac{\xi}{2} B^\alpha - \partial^i A_i^\alpha \right], \\ \tilde{c}^{\alpha*} &= \frac{\delta \Psi^C}{\delta c^\alpha} = 0, \\ \tilde{B}^{\alpha*} &= \frac{\delta \Psi^C}{\delta B^\alpha} = \frac{\xi}{2} \bar{c}^\alpha, \quad i = 1, 2, 3. \end{aligned} \quad (4.18)$$

$\xi$  is the gauge parameter in Coulomb gauge.  $Z^C$  is the generating functional for pure YM theory in Coulomb gauge in field/antifield formulation. Therefore, the FFBRST given by Eq. (3.2) with parameter given in Eq. (4.12) connects the generating functional  $Z^L$  to  $Z^C$ . Hence, the extended actions in Lorentz gauge  $W_\Psi^L(\phi, \phi^*)$  and in Coulomb gauge  $W_\Psi^C(\phi, \tilde{\phi}^*)$  which are the solutions of master equation are connected through FFBRST.

### Case III: Linear gauge to Quadratic gauge

Similarly, we choose another particular form of parameter  $\Theta(A, c, \bar{c}, B)$  related to

$$\Theta'(A, c, \bar{c}, B) = i \int d^4y \bar{c}^\alpha \left[ \gamma_1 \lambda B^\alpha - d^{\alpha\beta\gamma} A_\mu^\beta A^{\mu\gamma} \right], \quad (4.19)$$

and perform FFBRST transformation to  $Z^L$  as given in Eq. (4.1). The effective action  $S_{eff}[\phi]$  in  $Z^L$  will be modified due to the non-trivial Jacobian of these FFBRST transformation as  $S_{eff}[\phi] + S_1[\phi]$ , where

$$S_1 = \int d^4x \left[ \gamma_1 \lambda (B^\alpha)^2 - B^\alpha d^{\alpha\beta\gamma} A_\mu^\beta A^{\mu\gamma} - 2d^{\alpha\beta\gamma} \bar{c}^\alpha (D_\mu c)^\beta A^{\mu\gamma} \right]. \quad (4.20)$$

$d^{\alpha\beta\gamma}$  is a structure constant symmetric in  $\beta$  and  $\gamma$ . This additional term will transform the generating functional to

$$Z'^L = \int D\phi \exp \left\{ i \left[ W_\Psi^L(\phi, \phi^*) + S_1 \right] \right\} \equiv Z^Q. \quad (4.21)$$

This  $Z^Q$  is the generating functional for pure YM theory in quadratic gauge and is expressed in field/antifield formulation as

$$Z^Q = \int [dA dc d\bar{c} dB] \exp \left[ i \int d^4x \left\{ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + \tilde{A}^{\mu\alpha*} D_\mu^{\alpha\beta} c^\beta + \tilde{c}^{\alpha*} \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma + B^\alpha \tilde{c}^{\alpha*} \right\} \right]. \quad (4.22)$$

Or, in compact notation

$$Z^Q = \int D\phi \exp \left[ i S_0(\phi) + i \delta \Psi^Q \frac{1}{\Lambda} \right] \equiv \int D\phi \exp \left[ i W_\Psi^Q(\phi, \tilde{\phi}^*) \right], \quad (4.23)$$

$W_\Psi^Q(\phi, \tilde{\phi}^*)$  is the extended action in quadratic gauge and

$$\Psi^Q = \int d^4x \bar{c}^\alpha \left[ \frac{\xi}{2} B^\alpha - (\partial \cdot A^\alpha + d^{\alpha\beta\gamma} A_\mu^\beta A^{\mu\gamma}) \right], \quad (4.24)$$

with

$$\begin{aligned} \tilde{A}^{\mu\alpha*} &= \frac{\delta \Psi^Q}{\delta A_\mu^\alpha} = \partial^\mu \bar{c}^\alpha - 2d^{\alpha\beta\gamma} \bar{c}^\beta A^{\mu\gamma}, \\ \tilde{c}^{\alpha*} &= \frac{\delta \Psi^Q}{\delta \bar{c}^\alpha} = \left[ \frac{\xi}{2} B^\alpha - (\partial \cdot A^\alpha + d^{\alpha\beta\gamma} A_\mu^\beta A^{\mu\gamma}) \right], \\ \tilde{c}^{\alpha*} &= \frac{\delta \Psi^Q}{\delta c^\alpha} = 0, \\ \tilde{B}^{\alpha*} &= \frac{\delta \Psi^Q}{\delta B^\alpha} = \frac{\xi}{2} \bar{c}^\alpha. \end{aligned} \quad (4.25)$$

The FFBRST transformation with the parameter mentioned in Eq. (4.19), relates  $Z^L$  to  $Z^Q$ . Hence, the solutions of master equation are linked through FFBRST.

**Case IV: Lorentz gauge (with parameter  $\lambda$ ) to Lorentz gauge (with different parameter  $\lambda'$ )**

The discussion proceeds very much the same way as in earlier cases. We choose the FFBRST parameter

$$\Theta'(A, c, \bar{c}, B) = i\gamma \int d^4y \bar{c}^\alpha B^\alpha. \quad (4.26)$$

The extra term in the action for this case is calculated as

$$S_1 = \xi_1 \int d^4x (B^\alpha)^2. \quad (4.27)$$

The transformed generating functional is written in field/antifield formulation as

$$\begin{aligned} Z^{L(\lambda')} &= \int [dA dc d\bar{c} dB] \exp \left[ i \int d^4x \left\{ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + \tilde{A}^{\mu\alpha*} D_\mu^{\alpha\beta} c^\beta + \tilde{c}^{\alpha*} \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma + B^\alpha \tilde{c}^{\alpha*} \right\} \right], \\ &= \int D\phi \exp \left\{ i \left[ W_\Psi^{L(\lambda)}(\phi, \tilde{\phi}^*) + S_1 \right] \right\} \equiv \int D\phi \exp \left[ i W_\Psi^{L(\lambda')}(\phi, \tilde{\phi}^*) \right]. \end{aligned}$$

$W_\Psi^{L(\lambda')}(\phi, \tilde{\phi}^*)$  is the extended action in Lorentz gauge with gauge parameter  $\lambda'$  and

$$\Psi^{L(\lambda')} = \int d^4x \bar{c}^\alpha \left[ \frac{\lambda'}{2} B^\alpha - \partial \cdot A^\alpha \right], \quad (4.28)$$

with

$$\begin{aligned} \tilde{A}^{\mu\alpha*} &= \frac{\delta \Psi^{L(\lambda')}}{\delta A_\mu^\alpha} = \partial^\mu \bar{c}^\alpha, \\ \tilde{c}^{\alpha*} &= \frac{\delta \Psi^{L(\lambda')}}{\delta \bar{c}^\alpha} = \left[ \frac{\lambda'}{2} B^\alpha - \partial \cdot A^\alpha \right], \\ \tilde{c}^{\alpha*} &= \frac{\delta \Psi^{L(\lambda')}}{\delta c^\alpha} = 0, \\ \tilde{B}^{\alpha*} &= \frac{\delta \Psi^{L(\lambda')}}{\delta B^\alpha} = \frac{\lambda'}{2} \bar{c}^\alpha. \end{aligned} \quad (4.29)$$

Both the extended actions in field/antifield formulation,  $W_\Psi^L(\phi, \phi^*)$  and  $W_\Psi^{L(\lambda')}(\phi, \tilde{\phi}^*)$  are the solution of master equation given in Eq. (2.4) and are connected by FFBRST transformation.

## B. Connecting different solutions of master equation in effective theories

In this section, we show that not only the solutions of master equation in the theory with different gauges are connected through FFBRST but the solutions of master equation in different effective theories are also linked through FFBRST. In particular, the solutions of master equation in BRST invariant theory is related to the solution of master equation in through FFBRST. The generating functional for the most general BRST/anti-BRST invariant theory [17] is written as

$$Z_{AB}^L = \int [dA dc d\bar{c} dB] \exp \left[ i S_{eff}^{AB}(A, c, \bar{c}, B) \right], \quad (4.30)$$

where

$$\begin{aligned} S_{eff}^{AB}[A, c, \bar{c}, B] = \int d^4x \left[ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha - \frac{(\partial \cdot A^\alpha)^2}{2\xi} + \partial^\mu \bar{c} D_\mu c \right. \\ \left. + \frac{\alpha}{2} g f^{\alpha\beta\gamma} \partial \cdot A^\alpha \bar{c}^\beta c^\gamma - \frac{1}{8} \alpha (1 - \frac{1}{2} \alpha) \xi g^2 f^{\alpha\beta\gamma} \bar{c}^\beta \bar{c}^\gamma f^{\alpha\eta\xi} c^\eta c^\xi \right]. \end{aligned} \quad (4.31)$$

This effective action has the global symmetries under the following transformations.

BRST:

$$\begin{aligned} \delta A_\mu^\alpha &= (D_\mu c)^\alpha \Lambda, \\ \delta c^\alpha &= -\frac{1}{2} g f^{\alpha\beta\gamma} c^\beta c^\gamma \Lambda, \\ \delta \bar{c}^\alpha &= \left( \frac{\partial \cdot A^\alpha}{\xi} - \frac{1}{2} \alpha g f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma \right) \Lambda, \end{aligned} \quad (4.32)$$

anti-BRST:

$$\begin{aligned} \delta A_\mu^\alpha &= (D_\mu \bar{c})^\alpha \Lambda, \\ \delta c^\alpha &= -\frac{1}{2} g f^{\alpha\beta\gamma} \bar{c}^\beta \bar{c}^\gamma \Lambda, \\ \delta \bar{c}^\alpha &= \left( -\frac{\partial \cdot A^\alpha}{\xi} - (1 - \frac{1}{2} \alpha) g f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma \right) \Lambda. \end{aligned} \quad (4.33)$$

The most general BRST/anti-BRST effective action can be re-expressed in terms of aux-

iliary field  $B^\alpha$  as

$$S_{eff}^{AB}[A, c, \bar{c}, B] = \int d^4x \left[ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + \frac{\xi}{2} (B^\alpha)^2 - B^\alpha (\partial \cdot A^\alpha - \frac{\alpha g \xi}{2} f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma) \right. \\ \left. + \partial^\mu \bar{c} D_\mu c - \frac{1}{8} \alpha \xi g^2 f^{\alpha\beta\gamma} \bar{c}^\beta \bar{c}^\gamma f^{\alpha\eta\xi} c^\eta c^\xi \right]. \quad (4.34)$$

The off-shell nilpotent, global BRST/anti-BRST symmetries for the above effective action are given as

BRST:

$$\begin{aligned} \delta A_\mu^\alpha &= (D_\mu c)^\alpha \Lambda, \\ \delta c^\alpha &= -\frac{1}{2} g f^{\alpha\beta\gamma} c^\beta c^\gamma \Lambda, \\ \delta \bar{c}^\alpha &= B^\alpha \Lambda, \\ \delta B^\alpha &= 0. \end{aligned} \quad (4.35)$$

anti-BRST:

$$\begin{aligned} \delta A_\mu^\alpha &= (D_\mu \bar{c})^\alpha \Lambda, \\ \delta \bar{c}^\alpha &= -\frac{1}{2} g f^{\alpha\beta\gamma} \bar{c}^\beta \bar{c}^\gamma \Lambda, \\ \delta c^\alpha &= (-B^\alpha - g f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma) \Lambda, \\ \delta B^\alpha &= -g f^{\alpha\beta\gamma} B^\beta \bar{c}^\gamma \Lambda. \end{aligned} \quad (4.36)$$

Now, we choose a finite BRST parameter

$$\Theta'(A, c, \bar{c}, B) = i \int d^4y \bar{c}^\alpha \left[ \gamma_1 \lambda B^\alpha - \frac{1}{2} \alpha g \xi f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma \right], \quad (4.37)$$

and perform the FFBRST transformation given by Eq. (3.2) to  $Z^L$  written in Eq. (4.1).

We calculate the Jacobian corresponding to this FFBRST transformation and expressed as  $e^{iS_1(\phi)}$ , where

$$S_1 = \int d^4x \left[ \gamma_1 \lambda B^{\alpha 2} + \frac{1}{2} \alpha g \xi f^{\alpha\beta\gamma} B^\alpha \bar{c}^\beta c^\gamma - \frac{1}{8} \alpha \xi g^2 f^{\alpha\beta\gamma} \bar{c}^\beta \bar{c}^\gamma f^{\alpha\eta\xi} c^\eta c^\xi \right]. \quad (4.38)$$

This will give rise to a new generating functional

$$Z'^L = \int D\phi \exp \left\{ i \left[ W_\Psi^L(\phi, \phi^*) + S_1 \right] \right\}, \equiv Z_{AB}^L. \quad (4.39)$$

Under the choice of FFBRST parameter given in Eq. (4.37),  $Z^L$  transforms to  $Z_{AB}^L$  which can be expressed explicitly in field/antifield formulation as

$$Z_{AB}^L = \int [dA dc d\bar{c} dB] \exp \left[ i \int d^4x \left\{ -\frac{1}{4} F^{\alpha\mu\nu} F_{\mu\nu}^\alpha + \tilde{A}^{\mu\alpha*} D_\mu^{\alpha\beta} c^\beta - \tilde{c}^{\alpha*} \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma + B^\alpha \tilde{c}^{\alpha*} \right\} \right]. \quad (4.40)$$

In compact notation,

$$Z_{AB}^L = \int D\phi \exp \left[ i S_0(\phi) + i \delta \Psi_{AB}^L \frac{1}{\Lambda} \right] \equiv \int D\phi \exp \left[ i W_{AB}^L(\phi, \tilde{\phi}^*) \right], \quad (4.41)$$

where

$$\Psi_{AB}^L = \int d^4x \bar{c}^\alpha \left[ \frac{\xi}{2} B^\alpha - \left( \partial \cdot A^\alpha - \frac{\alpha g \xi}{4} f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma \right) \right], \quad (4.42)$$

with

$$\begin{aligned} \tilde{A}^{\mu\alpha*} &= \frac{\delta \Psi_{AB}^L}{\delta A_\mu^\alpha} = \partial^\mu \bar{c}^\alpha, \\ \tilde{c}^{\alpha*} &= \frac{\delta \Psi_{AB}^L}{\delta \bar{c}^\alpha} = \left[ \frac{\xi}{2} B^\alpha - \left( \partial \cdot A^\alpha - \frac{\alpha g \xi}{2} f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma \right) \right], \\ \tilde{c}^{\alpha*} &= \frac{\delta \Psi_{AB}^L}{\delta c^\alpha} = \frac{1}{4} \alpha g \xi f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma, \\ \tilde{B}^{\alpha*} &= \frac{\delta \Psi_{AB}^L}{\delta B^\alpha} = \frac{\xi}{2} \bar{c}^\alpha. \end{aligned} \quad (4.43)$$

$\xi$  is the gauge parameter in the most general BRST/anti-BRST invariant theory. Therefore, the FFBRST given by Eq. (3.2) with the parameter given in Eq. (4.37) takes  $Z^L = \int D\phi \exp [i W_\Psi^L(\phi, \phi^*)]$  to  $Z_{AB}^L = \int D\phi \exp [i W_{AB}^L(\phi, \tilde{\phi}^*)]$ .

Both the extended action,  $W_\Psi^L(\phi, \phi^*)$  and  $W_{AB}^L(\phi, \tilde{\phi}^*)$  in field/antifield formulation are the solution of master equation.

## V. CONCLUSION

In this paper, we explore the role of FFBRST transformation in field/antifield formulation. In field/antifield formulation the solutions of quantum master equation,  $W_\Psi[\phi, \phi^*]$  does not depend on the choice of  $\Psi$ , the gauge fixed fermion. We have considered FFBRST transformation in the field/antifield formulation of YM theories and have



shown that it connects the generating functional, corresponding to different solutions of master equation by considering several explicit examples. Particularly, we have shown that the generating functional  $Z^L$  in Lorentz gauge corresponding to the solution of master equation,  $W_\Psi^L[\phi, \phi^*]$  is connected to (i) generating functional  $Z^A$  corresponding to the solution of master equation in axial gauge  $W_\Psi^A[\phi, \phi^*]$ . (ii) generating functional  $Z^C$  corresponding to the solution of master equation in Coulomb gauge,  $W_\Psi^C[\phi, \phi^*]$ . (iii) generating functional  $Z^Q$  corresponding to the solution of master equation in quadratic gauge,  $W_\Psi^Q[\phi, \phi^*]$  and, (iv) generating functional  $Z^{L(\lambda')}$  corresponding to the solution of master equation in different gauge parameter,  $W_\Psi^{L(\lambda')}[\phi, \phi^*]$ . FFBRST not only connects theories with different solutions of master equations in different gauges but also it connects different  $W_\Psi[\phi, \phi^*]$  corresponding to different effective theories.  $W_\Psi^L[\phi, \phi^*]$  in BRST invariant theory is connected to  $W_{AB}^L[\phi, \phi^*]$ , corresponding to the most general BRST/anti-BRST invariant theory through FFBRST. In all the cases the non-trivial Jacobians of FFBRST play the crucial role.

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